PATTERNS AND RELATIONSHIPS 9 –12

Patterns and Relationships
What All Students "Should Know"
9 – 12

1. Mathematical patterns and relationships may be represented in various forms.

Clarifications:
All students should know various ways to represent patterns and relationships including:

- Numerical
- Verbal
- Graphical
- Symbolic
- Pictorial

2. Mathematical symbols can be used to represent real-world situations.

Clarifications:
All students should know real-world situations can be translated into:

- Numeric representations.
- Geometric representations.
- Algebraic representations.

3. Definitions of sequences and series.

Clarifications:
All students should recognize a (an):

- Arithmetic sequence: 1, 5, 9, 13
- Geometric sequence: 3, 15, 75, 375
- Arithmetic series: 1 + 5 + 9 + 13 + 17
- Geometric series: 3 + 15 + 75 + 375

4. Trigonometric ratios of sine, cosine, and tangent.

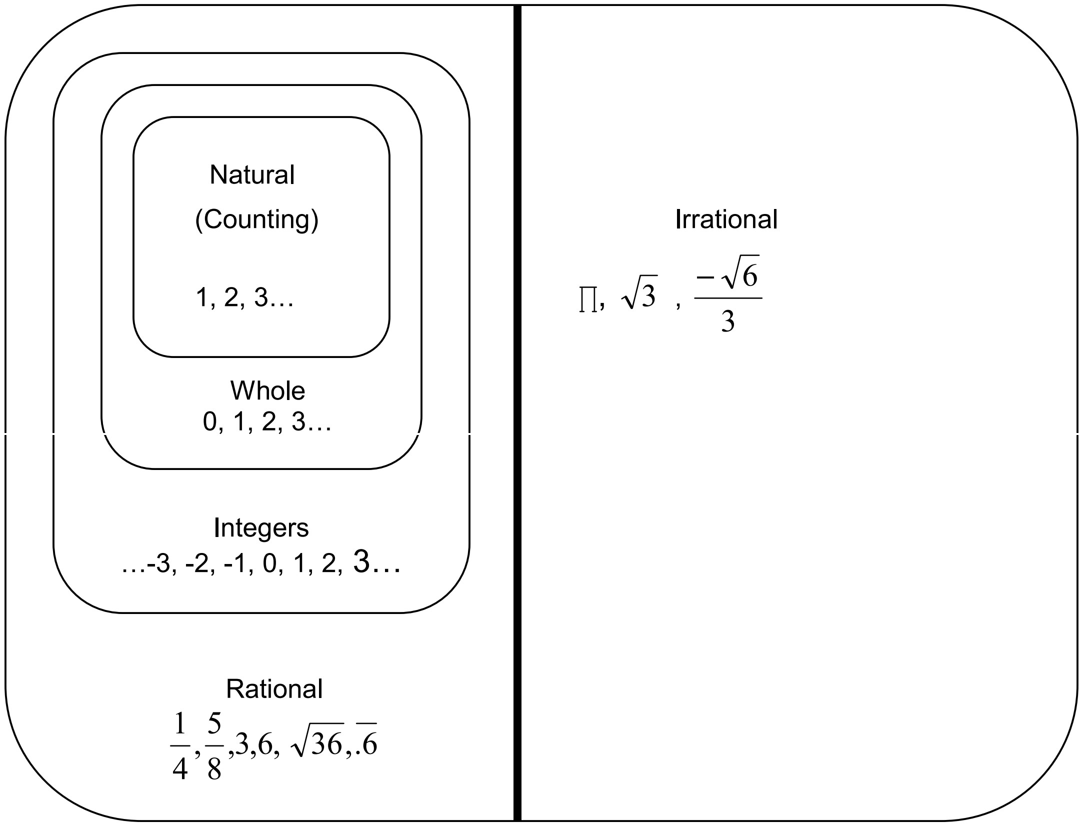
Clarifications:
All students should know and be able to use:

- Right triangle relationships.
- Pythagorean Theorem.

5. Subsets of the real number system.

Clarifications:

All students should identify the subsets of the real number system from a diagram similar to the illustration below.



Patterns and Relationships
What All Students "Should Do"
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Written Benchmark: A
Compare and contrast the real number system and its various subsystems with regard to their structural characteristics.

Problem 1:
Process Standards: 1.6, 3.2, 3.5, 3.7, and 4.1

Design a model that shows the connections and relationships between the following number sets: real, rational, irrational, integer, whole, and natural. This model should demonstrate an understanding of how these sets interrelate. Your model should include definitions of each set; examples of numbers contained in the set; examples of occupations that use these types of numbers. Your model should not be a Venn diagram.

Solution Notes:
Answers will vary.

Prerequisites:
Students should:
1. Know the subsets of the real number system.
2. Know the properties of real numbers.

Problem 2:
Process Standards: 1.4, 1.6, 2.1, and 3.5

[a $ b] is defined as $(a+2)*(b+3)$

A. What is the value of [a $ b] if a = 7, and b = ⅔. Show how you arrived at your solution.

A. If the value of [a $ b] = -6 and a = 3, determine the value of b. Show how you arrived at your solution.

B. Is a $ b commutative? i.e. Is [a $ b] = [b $ a]

Solution Notes:

A. 33 $(7+2)*(\frac{2}{3}+3)$

$= 9*3\frac{2}{3} = 27+6 = 33$

B. $-4\frac{1}{5}$ $(3+2)*(b+3)$ = -6

$5*(b+3)$ = -6

(b+3) = $\frac{-6}{5}$

b = $\frac{-6}{5}+(-3) = \frac{-6}{5}+\frac{-15}{5}$

b = $-\frac{21}{5}$ or $-4\frac{1}{5}$

C. [7 $ 2] ? [2 $ 7]
=
[7 $ 2] = (7=2) * (2+3)
= 14 x 5 = 60
[2 $ 7] = (2 + 2) x (7 + 3)
= 4 x 10 = 40
No it is not commutative.

Prerequisites:
Students should:
1. Know the subsets of the real number system.
2. Know the properties of real numbers.

Problem 3:
Process Standards: 1.4, 1.6, 2.1, and 3.5

If $\frac{p}{r}$ is an integer, determine which of the following must be integers.

A. 2p + 4r

B. $\frac{r}{p}$

C. $\frac{3p}{r}$

Solution Notes:

A. If p is $\frac{2}{3}$ and r is $\frac{1}{3}$, then 2p + 4r is not an integer.

B. If p is $\frac{2}{3}$ and r is $\frac{1}{3}$, then $\frac{r}{p}$ will not be an integer.

C. Regardless of what p or r is if $\frac{p}{r}$ is an integer, then $\frac{3p}{r}$ will also be an integer only three times larger.

Prerequisites:

Students should:

1. Understand the number sets of integers and rational numbers.
2. Know the properties of real numbers.

Patterns and Relationships
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Written Benchmark: B
Represent and analyze relationships using verbal rules, tables, and graphs as tools to interpret expressions, equations and inequalities.

Problem 1:
Process Standards: 1.6 and 1.8

Rock City charges $50.00 for court costs plus $3.00 for every mile per hour over the speed limit of 30 miles per hour. Stoneville charges $20.00 for court costs plus $5.00 for every mile per hour over the speed limit of 30 miles per hour. Complete the table. At what speed are the charges equal?

Miles Per Hour	Rock City	Stoneville
30 mph		

Miles Per Hour	Rock City	Stoneville
30 mph	0	0
32 mph	$56.00	$30.00
34 mph	$62.00	$40.00
36 mph	$68.00	$50.00
38 mph	$74.00	$60.00
40 mph	$80.00	$70.00
42 mph	$86.00	$80.00
44 mph	$92.00	$90.00
46 mph	$98.00	$100.00
48 mph	$104.00	$110.00
45 mph	**$95.00**	**$95.00**

The charges are the same when the motorist is traveling 45 mph in a 30 mph zone in both cities. (Note: Table can be shown with changes in mph column in other units of change).

Prerequisites:
Students should:

1. Be able to use tables to model functions.

Problem 2:

Process Standards: 1.6 and 1.8

While traveling down the Interstate Sammy decided to see how fast his mom was driving. He could not see the speedometer, but he could clock the time to go one mile (using the mileage markers on the Interstate). Sammy discovered that they traveled one mile in 47.22 seconds. How fast was Sammy's mom driving? Was she speeding if the speed limit was 70 mph? Show your work.

Solution Notes: $\frac{1 mile}{47.22s} \times \frac{60 \sec}{\min} \times \frac{60 \min}{hr} \approx 76mph$ so yes his mom is speeding!

Prerequisites:
Students should know:

1. Be able to use algebraic thinking to solve real-world problems.

Problem 3:
Process Standards: 1.6 and 1.8

Given the following equations, answer the questions listed below:

$$ax + 3y = 9$$
$$2x + 4y = c$$

A. Find a and c if these two equations have the point (3, 1) in common.

B. Find a and c if these two equations are parallel.

C. Find a and c if these two equations are perpendicular.

Solution Notes:
A. a = 2 and c = 10;
B. a = 1.5 and c can equal any real number but 12; and
C. a = -6 and c can be any real number.

Prerequisites:
Students should:
1. Understand the relationships of comparing slopes of lines.
2. Know the properties of parallel and perpendicular slopes.

Problem 4:
Process Standards: 1.6 and 1.8
Find the common solution (x, y) for the two equations below.

$$2x - 3y = 7$$
$$3x + y = 5$$

Solution Notes:

$$2x - 3y = 7 --> 2x - 3y = 7$$
$$3x + y = 5 --- > 9x + 3y = 15 --- > 11x = 22$$
$$x = 2$$

$$2(2) - 3y = 7$$ solution: (2, -1)
$$4 - 3y = 7$$
$$-3y = 3$$
$$y = -1$$

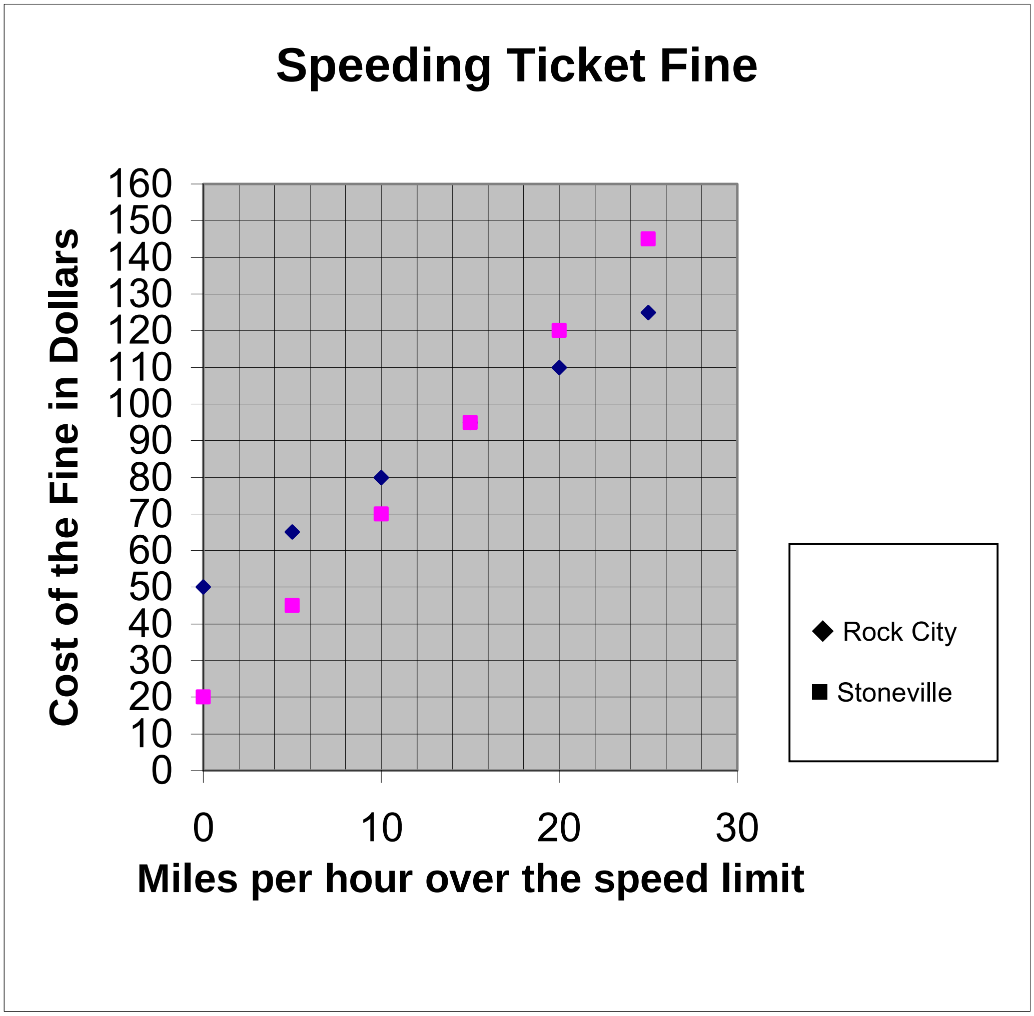
Problem 5:
Process Standards: 1.6 and 1.8

Using the following table, describe three or more patterns and give a rule that shows the data in an equation form.

x	y
2	10
3	20
4	34
5	52
6	74

Solution Notes:
Answers may vary, but might include the following patterns: even answers (y values); increasing changes; each y-value is 2 more than a multiple of the x-value; the changes are 2*5 then 2*7, 2*9, and 2*11; the multiples listed above are one more than the number doubled. Equation should be $y = 2x^2 + 2$.

<u>Problem 6</u>:
Process Standards: 1.6 and 1.8



A. How many mph over the posted speed limit are the fines equal for both cities?
B. Determine the fines in each city if you are traveling 20 mph over the limit.
C. John draws a line that connects the points for each city. What is wrong with his method?
D. Discuss the cost of speeding at zero miles per hour over the speed limit.

Solution Notes:

A. 15 mph
B. Rock City $110 and Stoneville $120
C. The graph is not continuous since the data are specific values by whole miles over the speed limit.

Prerequisites:

All students should know:

1. How to graph discrete data.

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Written Benchmark: C

Translate among tabular, symbolic, and graphical representations of functions and model real-world phenomena with a variety of functions.

Problem 1:
Process Standards: 1.8, 3.5, and 3.6

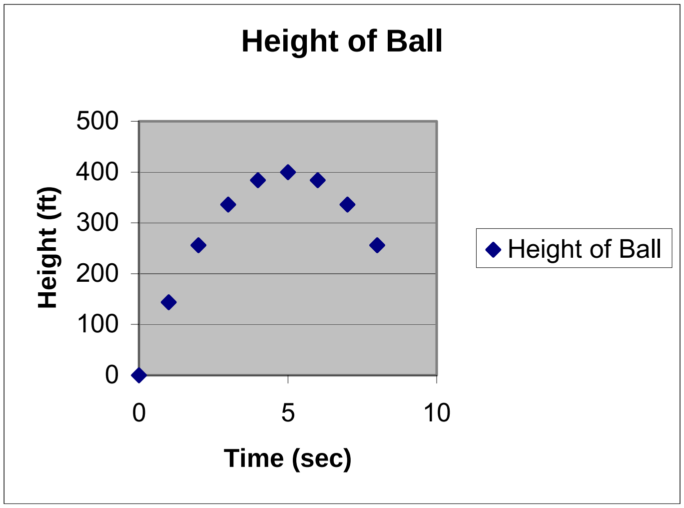
Use a Graphing Calculator and/or computer spreadsheet to solve problems using the formula $-16t^2 + 160t = h(t)$. Where h(t) represents the height of the object in feet at a given time (t) in seconds.

A. Construct a table of data of heights for time periods of 0 to 8 seconds.
B. Make a graph of the data in your table.
C. After how many seconds will the object reach its maximum height?
D. At what time will the object return to the ground?

Solution Notes:
A.

Time (sec)	Height (ft)
0	0
1	144
2	256
3	336
4	384
5	400
6	384
7	336
8	256

B.



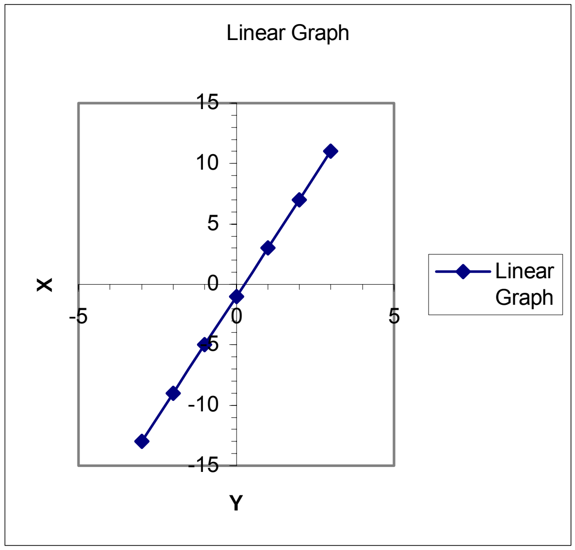
C. 5 sec.
D. 10 sec.

<u>Problem 2</u>:
Process Standards: 1.6 and 1.8

A. Graph the data in the table below.
B. Express the relationship between the numbers in column *x* and in column *y* in the form of an equation. (x and y are elements of the real number set.)

x	y
-3	-13
-2	-9
-1	-5
0	-1
1	3
2	7
3	11

Solution Notes:
A.



B. $y = 4x - 1$

Prerequisites:
Students should:

1. Demonstrate the ability to graph on a coordinate plane.
2. Write an equation given data.

Problem 3:
Process Standards: 1.4, 1.6, 3.5, and 3.6

The Junior Class has decided to sell balloons for Valentine's Day and use the profit for Prom expenses. The balloons they selected cost $0.29 each. The Prom Committee wants to determine how many balloons are needed and the amount to charge for each balloon in order to maximize the profit. The committee discovered that 950 people would purchase a balloon if they cost $1.00 each. However for every $0.25 the Junior class charges per balloon, 100 fewer people will purchase one.

A. Create a table of data, using a graphing calculator or computer spreadsheet, which represents the information provided above, and the profit predicted.
B. Graph the data from the table.
C. Write an equation that will represent the information presented above.
D. What should the Junior Class charge per balloon? Defend your decision mathematically.

Solution Notes:
A. Spreadsheet screen of the initial data for the problem.
B. Graph screen of the spreadsheet.
C. $P = (950 - 100x)(1.00 + .25x) - (950 - 100x)(.29)$
D. $P = (950 - 100x)(.71 + .25x)$ or $P = -25x^2 + 166.5x + 674.5$
E. To maximize the profit made by the junior class the price per balloon should be $1.75.

Balloons	Selling price per Balloon	Income	Cost of the Balloons	Profit
950	$1.00	$950.00	$275.50	$674.50
850	$1.25	$1,062.50	$246.50	$816.00
750	$1.50	$1,125.00	$217.50	$907.50
650	$1.75	$1,137.50	$188.50	$949.00
550	$2.00	$1,100.00	$159.50	$940.50
450	$2.25	$1,012.50	$130.50	$882.00
350	$2.50	$875.00	$101.50	$773.50
250	$2.75	$687.50	$72.50	$615.00
150	$3.00	$450.00	$43.50	$406.50
50	$3.25	$162.50	$14.50	$148.00

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Written Benchmark: D

Represent situations that involve variable quantities with expressions, equations, and inequalities.

Problem 1:

Process Standards: 1.8, 3.5, and 3.6

The animal trainer at the zoo tells you that each male lion consumes 26 pounds of raw meat per day, and females consume 19 pounds per day.

A. If the zoo currently has 3 male lions and 5 female lions, how many pounds of raw meat are needed for a day?
B. Translate the above information into a linear equation to find the amount of food needed for a week with (m) male and (f) female lions.

Solution Notes:
No solution.

Prerequisites:
Students should:

1. Know how to evaluate expressions.
2. Know how to represent situations algebraically.

Problem 2:

Process Standards: 1.8, 3.5, and 3.6

Pete is going to race his twin sister, Pam, on a track. Since both Pete and Pam agree that she is faster, they will try to make the race fair by giving Pete a head start. They have marked off the 100-meter course with Pam beginning at the starting line and Pete beginning at the 20-meter mark. If Pam can run at a rate of 10.1 meters per second and Pete runs at a rate of 7.9 meters per second, who will win the race? Justify your selection.

Solution Notes:
Solutions may vary from graphical representations to using equations. The two equations that would represent the data would be $d = 10.1t$ for Pam and $d = 7.9t + 20$ for Pete. Using these two equations students should see the intersection (where Pam catches Pete) at about 91 meters just before the finish line, which means that she would win.

Prerequisites:
Students should:
1. Know how to evaluate expressions.
2. Know how to represent situations algebraically.

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Written Benchmark: E

Solve equations and inequalities.

Problem 1:

Process Standards: 1.5 and 1.8

Pat is preparing to roast a turkey. The cooking chart states that at 400º the cooking time (t) in minutes is dependent upon the weight (p) of the turkey. For a turkey weighing 10 pounds and under, the time is determined by: $t \geq 16p + 20$ and for a turkey weighing more than 10 pounds the time is determined by: $t \geq 18p$. Sketch a graph of these inequalities. How long will it take for a 16-pound turkey to cook? If a 13-pound turkey has been cooking for 3 hr. 5 min., how much longer does it need to cook?

Solution Notes:

A. Graph from the graphing calculator screen that was created for $y \geq 16x + 20$ for $x < 10$ and $y \geq 18x$ for $x \geq 10$)

B. At least 4 hr. 48 min. $y = 18(16) = 288$ min.

C. 49 min. 3 hr. 54 min.

<u>3 hr. 5 min</u>.

49 min.

Prerequisites:

Students should:

1. Know how to graph inequalities.

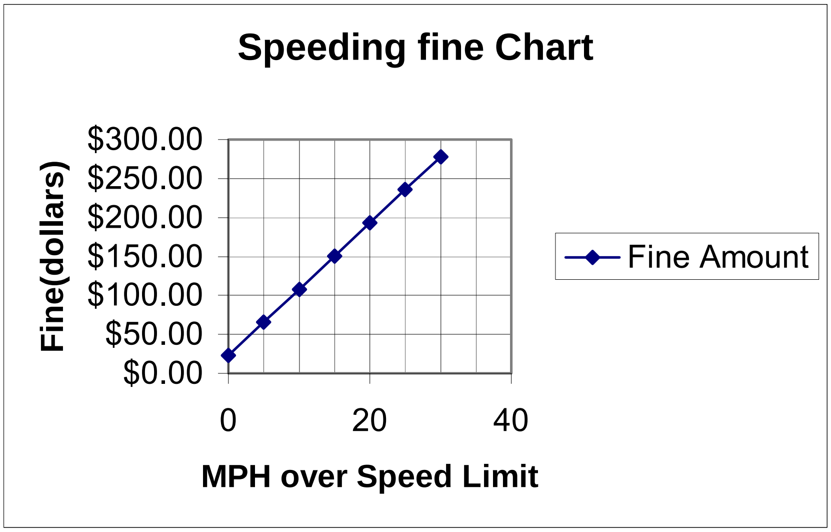
Problem 2:

Process Standards: 1.6 and 1.10

A. The highway patrolman determines the amount owed for a traffic ticket by using the formula: $F = 23 + 8.5(x - 70)$, where F represents the fine and x represents the speed of the vehicle. How fast was the speeder traveling if the fine is $125.00?

B. A second patrolman uses the following graph to determine the fine for speeding. How does this compare to the solution in part A?

C. Which method would be most efficient? Justify your answer.



D. The equation in part (A) appears to vary from the graph in part (B). Examine the patterns and relationships then compare the similarities and differences in the information each part provides.

Solution Notes:

A. $125 = 23 + 8.5\,(x - 70) \rightarrow 102 = 8.5\,(x - 70) \rightarrow 12 = x - 70 \rightarrow 82 = x$

B. Answers should be approximately the same.

C. Answers may vary.

D. Answers should include recognition that these are two ways to represent the information (formula) differently and explain why.

Prerequisites:

All students should know:

1. How to solve equations.

<u>Problem 3</u>:

Process Standards: 1.6 and 1.10

Solve $x - 5 > 8$. Graph the solution on a number line.

Problem 4:
Process Standards: 1.6 and 1.10

Solve $-3x < x + 8$. Graph the solution on a number line.

Problem 5:
Process Standards: 1.6 and 1.10

Solve for y: $3x - y = 2y + 9$. Graph the solution in a coordinate plane. What is the slope and y- intercept of the graph.

Problem 6:
Process Standards: 1.6 and 1.10

Solve the system of equations:

$$-x + 2y = -2$$
$$y = \frac{1}{2}x + 3$$

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Written Benchmark: F

Translate between synthetic and coordinate representation for geometric relationships.

<u>Problem 1</u>:

Process Standards: 3.3 and 3.5

The following coordinates are the vertices of a quadrilateral:
A (5,5)
B (3, -4)
C (-5, -4)
D (-1,3)

Is this quadrilateral a parallelogram? Show your work to justify your answer mathematically. Explain your reasoning in words.

Solution Notes:

$$m = \frac{5-(-4)}{5-3} = \frac{9}{2}$$

for A(5, 5); B(3, -4)

$$m = \frac{-4-(-4)}{-5-3} = \frac{0}{-8}$$

for B(3, -4); C (-5, -4)

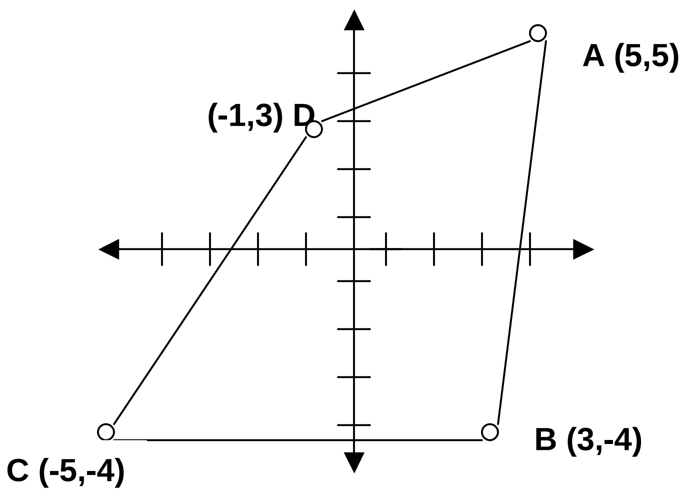
$$m = \frac{-4-3}{-5-(-1)} = \frac{-7}{-4} = \frac{7}{4}$$

for C(-5,-4); D(-1,3)

$$m = \frac{5-3}{5-(-1)} = \frac{2}{6} = \frac{1}{3}$$

for D (-1,3); A(5,5)

For the opposite sides of a quadrilateral to be parallel the slopes of the opposite sides must be equal 9/3 ≠ 7/4 and 0 ≠ 1/3 therefore the quadrilateral does not contain 2 sets of parallel lines, and is not a parallelogram.

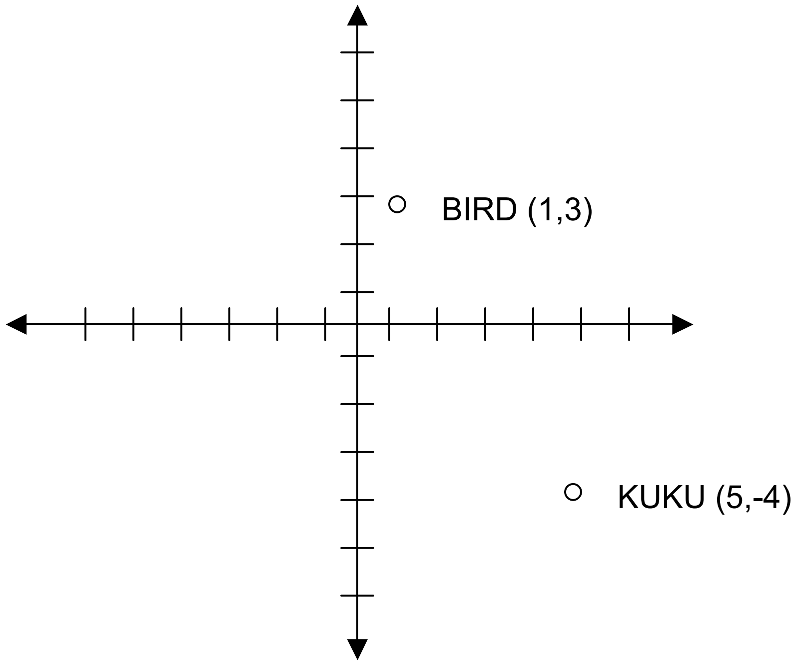


Some students would definitely consider this graph and a written explanation as having performed the required task: i.e., after graphing the points I can see that the opposite sides are not parallel because they do not have the same constant rate of change. Therefore, the quadrilateral is not a parallelogram.

Prerequisites:
Students should:
1. Know how to determine the slope of a line.
2. Be able to determine parallel lines.
3. Know the definition for a parallelogram.
4. Be able to use the connections these ideas provide.

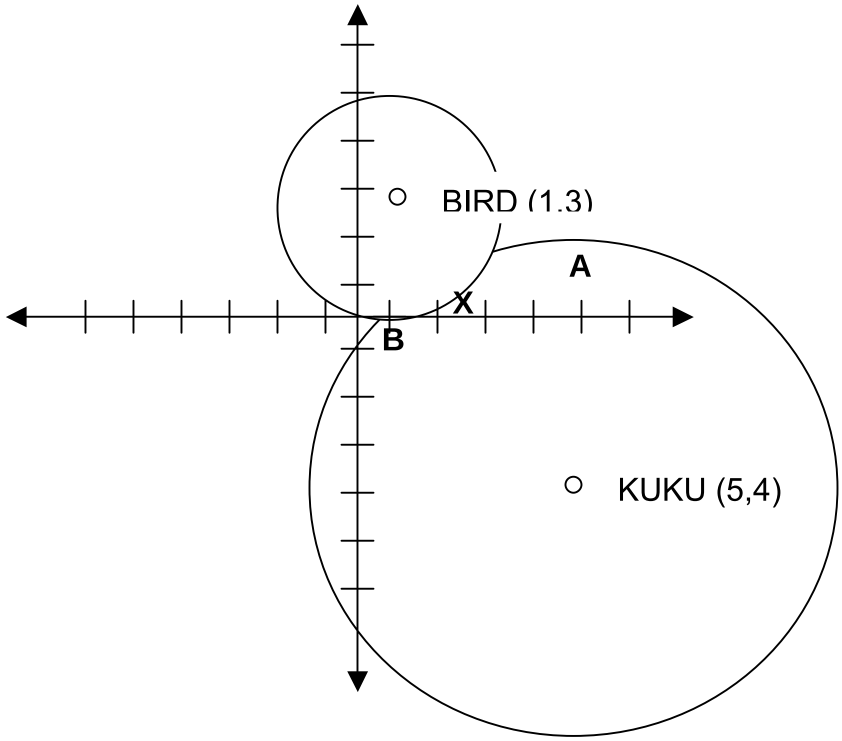
Problem 2:
Process Standards: 3.1, 3.5, and 3.6

Radio Stations KUKU broadcast to all points within a radius of 75 miles. Radio Stations BIRD broadcasts to all points within a radius of 60 miles. One unit = 15 miles.



A. On the coordinate plane, show the broadcast area of both stations.
B. Explain why these two stations could not have the same station number.

Solution Notes:
Radio Stations KUKU can broadcast to all points within a radius of 75 miles. Radio Stations BIRD broadcasts to all points within a radius of 60 miles. The coordinate plane shows the broadcast area of both stations and could also represent a grid system like longitude and latitude. Since their areas overlap they can not have the same station number.



Prerequisites:
Students should:
1. Know how to graph using the coordinate system.
2. Know the properties of circles.

Patterns and Relationships
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Written Benchmark: G
Investigate limiting processes by examining infinite sequences and series.

<u>Problem 1</u>:
Process Standards: 1.6, 3.5, and 3.6

Given the functions:
i. $y=x$
ii. $y=x^2$
iii. $y=x^3$
What boundaries for x would cause y to be within:
A. 63.75 and 64.25
B. 63.9 and 64.1
C. 63.95 and 64.05
D. 63.99 and 64.01
Use technology to investigate the following:
1. y = 1/x
   A. State the values of y if x = 1, x = 10, x = 100, x = 1000, x = 1,000,000,000.
   B. As x gets increasingly large, what value does y approach?

2. y = 1/ (x – 5)
State the values for y if x = 10, x = 6, x = 5.5, x = 5.25, x = 5.01, x = 5.001.
If x starts at 10 and gets closer to 5, what value does y approach?

Solution Notes:

	A	B	C	D
i. y = x	$63.75 \le x \le 64.25$	$63.9 \le x \le 64.1$	$63.95 \le x \le 64.05$	$63.99 \le x \le 64.01$
ii. $y = x^2$	$\sqrt{63.75} \le x \le \sqrt{64.25}$ 7.984 8.016	$\sqrt{63.9} \le x \le \sqrt{64.1}$ 7.994 8.006	$\sqrt{63.9} \le x \le \sqrt{64.05}$ 7.997 8.003	$\sqrt{63.99} \le x \le \sqrt{64.01}$ 7.999 8.0006
iii. $y = x^3$	$\sqrt[3]{63.75} \le x \le \sqrt[3]{64.25}$ 3.995 4.005	$\sqrt[3]{63.9} < x < \sqrt[3]{64.1}$ 3.998 4.002	$\sqrt[3]{63.95} < x < \sqrt[3]{64.05}$ 3.999 4.001	$\sqrt[3]{63.99} < x < \sqrt[3]{64.01}$ 3.9998 4.0002

1. A. (1,1) (10, .1) (100, . 01) (1000, .001) (1,000,000,000, .000000001)
   B. 0
2. A. (10, 1/5) (6,1) (5.5, 2) (5.25, 4) (5.01, 100)
   B. Infinity

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Written Benchmark: H

Apply trigonometry to problem situations involving triangles and explore real-world phenomena using the sine, cosine and tangent functions.

<u>Problem 1</u>:

Process Standards: 1.8, 3.5, and 3.6

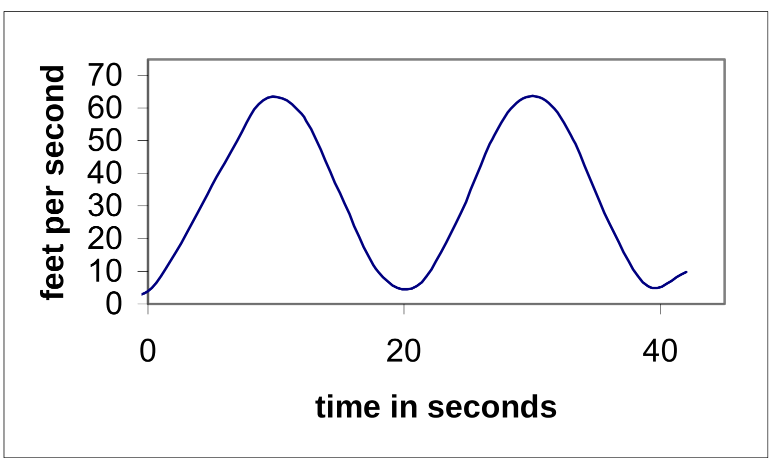
Jane is riding on a Ferris wheel at the state fair. She was the last person to be seated. When she enters her seat, she notices she is 4 feet above the ground. The Ferris wheel has a radius of 30 feet and makes 3 revolutions per minute.

A. Write a sinusoidal function that would model Jane's distance from the ground at any time (t).
B. Graph your model.
C. How long will it take before Jane reaches the maximum height for the first time? Justify your answer.
D. How far above the ground will Jane be 15 seconds after the ride begins? 28 seconds?

Solution Notes:

A. $h(t) = -30 \cos (\pi t/10) + 34$

B.



C. 10 Seconds – It takes 20 seconds to complete a revolution so it would take ½ of 20 seconds or 10 seconds to reach the top, or:
64 = -30 cos(תt/10) + 34
30 = -30 cos(תt/10)

Cos $^{-1}$ = תt/10
ת = תt/10
t = 10 seconds

D. y = h(t) = -30 cos (15ת/10) + 34
h(15) = 34 feet
h(28) = -39 cos(2.8ת) + 34 = 58.3 feet

Prerequisites:
All students should know:
1. How to graph trigonometric functions.

<u>Problem 2</u>:
Process Standards: 3.3 and 3.5

Pat's flowerbed is a triangular shape. One side of the triangle measures 7 feet, another side of the triangle measures 8 feet, and the third side of the triangle measures 3 feet. Is the flowerbed a right angle? Show the work that supports your decision.

Solution Notes:
The student should demonstrate that the sum of the squares of the two shorter sides does not equal the square of the longest side.

Teacher Note:
This problem was added to this section to remind students that the relationships we are working here work specifically with right triangles.

Prerequisites:
Students should:
1. Know how to use the Pythagorean theorem.

<u>Problem 3</u>:
Process Standards: 1.8 and 1.10

At Skyland Ski Resort two ski slopes are open today. The Red Slope is 1500 feet long and starts at 1200 feet above ground level. The Blue Slope starts at 1300 feet above ground level and is 2000 ft long. Show how you would determine the steepness of each slope. As a beginner, which slope would you choose and why?

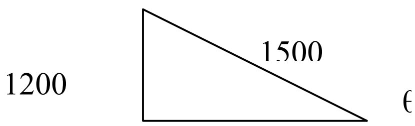
Solution Notes:

Red: $\sin\theta = \frac{1200}{1500}$ Blue: $\sin\lambda = \frac{1300}{2000}$

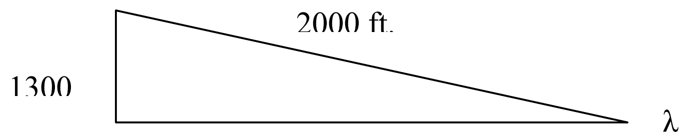
$\theta = 53.13^\circ$ $\lambda = 40.54^\circ$

Or Diagrams:

Red



Blue



I would choose the Blue slope because it is not as steep.

Prerequisites:

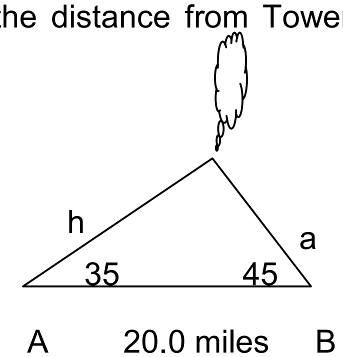
Students should:

1. Know how to determine the angle of inclination and the rates of change.

<u>Problem 4</u>:

Process Standards: 1.6 and 3.5

In Mark Twain National Forest, there are two fire towers 20.0 miles apart. The spotters in Tower A and B both see the column of smoke in the distance. The diagram below shows the angle measures from each tower to the column of smoke in the distance. Determine mathematically the distance from Tower A to the smoke and the distance from Tower B to the smoke.



Solution Notes:

$$\frac{\sin 100}{20} = \frac{\sin 35}{A} = \frac{\sin 45}{B}$$

A. = 11.6 therefore the distance from Tower B to the smoke is 11.6 miles.

B. = 14.4 therefore the distance from Tower A to the smoke is 14.4 miles.

Prerequisites:

Students should:

1. Know the Law of Sines.

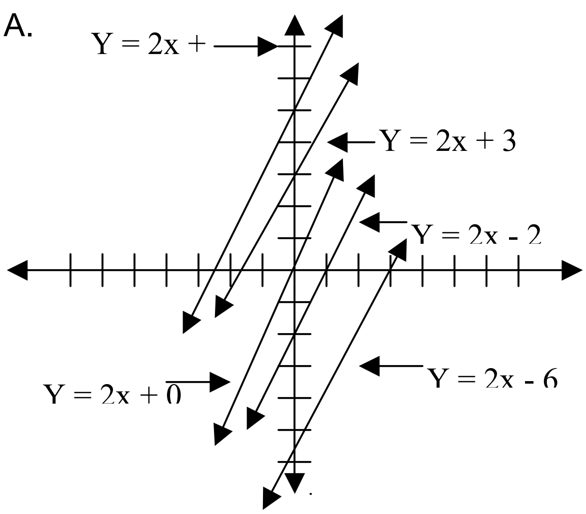
Patterns and Relationships
What All Students "Should Do"
9 – 12

Written Benchmark: I
Analyze effects of parameter changes on the graphs of functions using a variety of technologies to gather data.

<u>Problem 1</u>:
Process Standards: 1.6, 2.7 and 3.3

Enter the linear equation y = 2x + 1 into a graphing calculator or use graphing software. Sketch the graph. Enter 5 more linear equations changing only the constant term each time. Evaluate the effect of changing this constant on the graph of linear equations.

Solution Notes:
A.



B. The Calculator screen will show 5 linear functions with the y intercept changes.

Prerequisites:
Students should:
1. Know how to graph linear equations.

Problem 2:
Process Standards: 1.6, 2.7, and 3.3

Given an equation in the form y = ax + b, describe the following
(HINT: what is the impact to the graph):

A. What would happen to the graph if a in the equation is multiplied by c (making the equation y = c (ax) + b) if c > 0?

B. What would happen to the graph if a in the equation is multiplied by c (making the equation y = c (ax) + b) if c < 0?

Solution Notes:
Solutions may vary
IF c > 0 and a > 0

- then the slope will become steeper when c > 1 and less steep if c is between 0 and 1. (If the slope is steeper it means the line will go up faster (than it goes to the right) as one moves to the right along the line).

IF c > 0 and a < 0

- then the slope will become steeper when c > 1 and less steep if c is between 0 and 1. (If the slope is steeper it means the line will go down faster (than it goes to the right) as one moves to the right along the line).

IF c < 0 and a > 0

- then the slope will become steeper when c < -1 and less steep if c is between 0 and -1. (If the slope is steeper it means the line will go down faster (than it goes to the right) as one moves to the right along the line).

IF c < 0 and a < 0

- then the slope will become steeper when c < -1 and less steep if c is between 0 and -1. (f the slope is steeper it means the line will go up faster (than it goes to the right) as one moves to the right along the line).